An estimate for the location of QCD critical end point

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It is proposed that a study of the ratio of shear viscosity to entropy density $\frac{\eta}{s}$ as a function of the baryon chemical potential μ_B, and temperature T, provides a dynamic probe for the critical end point (CEP) in hot and dense QCD matter. An initial estimate from an elliptic flow excitation function gives $\mu_B^{\rm cep} \sim 150-180$ MeV and $T_{\rm cep} \sim 165-170$ MeV for the location of the the CEP. These values place the CEP in the range for "immediate" validation at RHIC.

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The phase boundaries and the critical end point (CEP) are fundamental characteristics of hot and dense nuclear matter [1]. The study of heavy ion collisions has been proposed [2] as an avenue to search for these essential characteristics of the Quantum Chromodynamics (QCD) phase diagram i.e. the plane of temperature vs baryon chemical potential (T, μ_B).

A recent resurgence of experimental interest in the CEP has been aided by strong experimental and theoretical evidence for a crossover transition to the quark gluon plasma (QGP) in heavy ion collisions at the Relativistic Heavy Ion collider (RHIC) [3, 4, 5, 6, 7, 8, 9, 10, 11]. Such a crossover, constitutes a necessary requirement, albeit insufficient, for locating the CEP.

Several attempts have been made to provide theoretical guidance on where to localize a search for the CEP in the QCD phase diagram [7, 12, 13, 14, 15]. The resulting predictions for the critical values of temperature $T_{\rm cep}$ and baryon chemical potential $\mu_B^{\rm cep}$, which locates the CEP, have not converged and now span a broad range.

Therefore, recent plans for the experimental verification of the CEP have centered on energy scans with an eye toward accessing the broadest possible range of μ_B and T values [16, 17, 18, 19]. Fig. 1 reenforces the value of such energy scans; it shows the chemical freezeout values of μ_B (top panel) and T, as a function of beam collision energy $\sqrt{s_{NN}}$, extracted via chemical fits to particle ratios [20] obtained at several accelerator facilities. This unprecedented reach in μ_B and T values, clearly indicate that the combined results from energy scans at the Facility for Anti-proton and Ion Research (FAIR), the Super Proton Synchrotron (SPS) and RHIC, will allow access to the full range of μ_B and T values necessary for a comprehensive CEP search.

At the CEP (or close to it) anomalies can occur in a wide variety of dynamic and static properties. Anomalies in dynamic properties reflect a change in quantities such as the transport coefficients and relaxation rates, multi-time correlation functions and the linear response

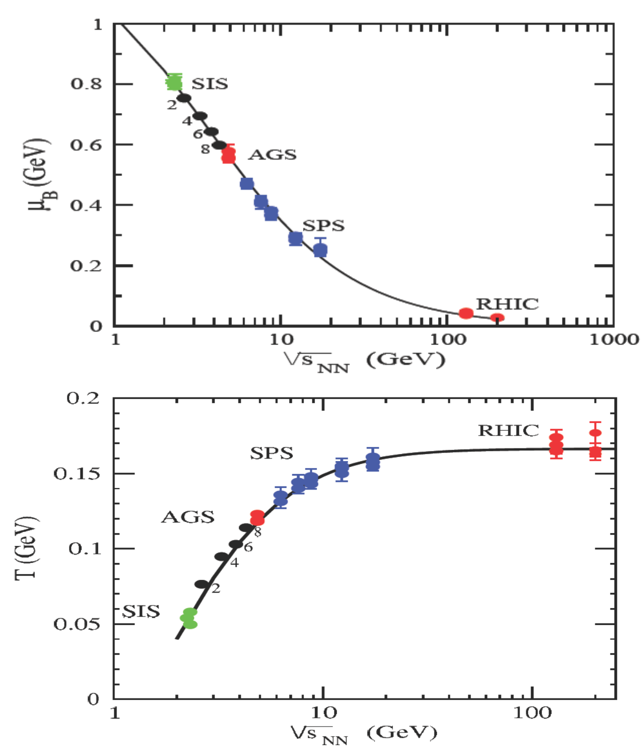

FIG. 1: (Color online) $\sqrt{s_{NN}}$ dependence of μ_B (top panel) and T obtained from chemical fits [20] to particle yield ratios obtained at different accelerator facilities as indicated. The solid lines are fits to these data.

to time-dependent perturbations. All of these depend on the equations of motion, and are not simply determined by the equilibrium distribution of the particles at a given instant of time. By contrast, static properties are solely determined by the single-time equilibrium distribution which include thermodynamic coefficients, single-time correlation functions, and the linear response to time-independent perturbations.

Critical fluctuations are thought to be one of the more

important static signals for locating the CEP [21]. Consequently, extensive studies of particle fluctuations have been made over a broad range of beam collision energies. To date, no definitive observation indicative of the CEP, has been reported. A comprehensive search for the CEP via dynamic variables is still lacking.

In this letter, we argue that it is possible to locate the CEP via study of the μ_B and T dependence of the ratio of viscosity to entropy density ($\frac{\eta}{s}$) [22, 23, 24] and give an estimate for $\mu_B^{\rm cep}$ and $T_{\rm cep}$ (i.e the location of the CEP) using existing data.

The rationale for using $\frac{\eta}{s}$ as a probe for the CEP is two fold. First, we observe that this QCD critical endpoint belongs to the universality class of the 3d Ising model, ie. the same universality class for a liquid-gas system; here, it is important to recall that all members of a given universality class have "identical" critical properties. Second, we observe that $\frac{\eta}{s}$, for atomic and molecular substances, exhibits a minimum of comparable depth for different isobars passing in the vicinity of the liquid-gas critical end point [22, 23, 24, 25]. Fig. 2 illustrates this for H_2O. When an isobar passes through the critical end point, the minimum forms a cusp at the reduced temperature $\frac{T-T_{\rm cep}}{T_{\rm cep}} = 0$; when it passes above the critical end point (i.e a pressure P above the critical pressure $P_{\rm cep}$), a less pronounced minimum is found at a value slightly above $\frac{T-T_{\rm cep}}{T_{\rm cep}} = 0$. For an isobar passing just below the critical pressure, the minimum is found at $\frac{T-T_{\rm cep}}{T_{\rm cep}} < 0$ (liquid side) but is accompanied by a discontinuous change across the phase transition. Thus, for a range of reduced temperatures, the average value $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ can be seen to grow rapidly for isobars passing through the critical end point and just below it. This is illustrated in the inset of Fig. 2 where $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ is plotted vs pressure for the reduced temperature range $\frac{T-T_{\rm cep}}{T_{\rm cep}} = 0 - 0.3$. Fig. 2 shows that the CEP is signaled by a minimum at $\frac{T-T_{\rm cep}}{T_{\rm cep}} \sim 0$, in the dependence of $4\pi(\frac{\eta}{s})$ on the reduced temperature, as well as a sharp increase in $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ vs P for $\frac{T-T_{\rm cep}}{T_{\rm cep}} \gtrsim 0$.

In analogy to the observations for atomic and molecular substances, one expects a range of trajectories, in the (T, μ_B) plane for decaying nuclear systems, to show $\frac{\eta}{s}$ minima with a possible cusp at the critical end point $(T_{\rm cep}, \mu_B^{\rm cep})$. That is, for $\mu_B = \mu_B^{\rm cep}$ the $\frac{\eta}{s}$ minimum is expected at the reduced temperature $\frac{T-T_{\rm cep}}{T_{\rm cep}} = 0$; for other values of μ_B with associated critical temperature T_c not too far from $T_{\rm cep}$, the dependence of $4\pi(\frac{\eta}{s})$ on $\frac{T-T_{\rm cep}}{T_{\rm cep}}$ is also expected to grow stronger as μ_B is increased from an initially small value up to $\mu_B \gtrsim \mu_B^{\rm cep}$. Indeed, recent calculations for different types of phase transitions (first-order, second-order and a crossover) suggest a rapid change in the value of η/s in the vicinity of the CEP [26].

For a given value of μ_B, a hot nuclear system for which $\frac{T-T_{\rm cep}}{T_{\rm cep}} > 0$ will sample the full range of $\frac{\eta}{s}$ values to give an average, as it evolves toward the $\frac{\eta}{s}$ minimum. Consequently, one expects an increase of $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ with increasing μ_B, punctuated by a relatively rapid increase for μ_B values slightly above $\mu_B^{\rm cep}$. The latter would be comparable to the rapid increase in $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ observed for H_2O (inset in Fig. 2) when P is lowered a little below the critical pressure.

Therefore, the extraction of $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ as a function of T and μ_B from experimental data, can serve as a constraint for the location of the CEP. Such extractions are possible from an elliptic flow excitation function measurement because a sizable change in $\left\langle 4\pi(\frac{\eta}{s})\right\rangle$ is expected to lead to a measurable suppression of the magnitude of elliptic flow. It could even serve to invalidate the currently observed universal scaling patterns [27, 28, 29].

In recent work [23], we have used elliptic flow measurements to obtain the estimates $\left\langle 4\pi(\frac{\eta}{s})\right\rangle \sim 1.3$ [30] and $\langle T\rangle \sim 165$ MeV for hot and dense matter [31] produced in Au+Au collisions ($\sqrt{s_{NN}} = 200$ GeV or $\mu_B \sim 24$ MeV) at RHIC. A comparison of this $\frac{\eta}{s}$ value to those calculated for a meson-gas for $T < T_c^Q$ [24], and the QGP for $T > T_c^Q$ ($T_c^Q \sim 170$ MeV [32]), gave a good indication for the expected minimum (for T close to $T_{\rm cep}$) in the plot of $4\pi(\frac{\eta}{s})$ vs $\frac{T-T_c^Q}{T_c^Q}$. We therefore use this observation as a basis for the estimate $T_{\rm cep} \sim 165 - 170$ MeV. This estimate is similar to the chemical freeze-out temperature for a broad range of collision energies (see botom panel of Fig. 1). This constancy of the freeze-out temperature ($T \sim 165$ MeV) may be a further indication that chemical freeze-out occurs at, or close to $T_{\rm cep}$ for $\sqrt{s_{NN}} \sim 17 - 200$ GeV.

The value $\left\langle 4\pi(\frac{\eta}{s})\right\rangle \sim 1.3$, achieved in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, is rather close to the conjectured lower bound of $4\pi(\frac{\eta}{s}) = 1.0$. Consequently, one can conclude that, for $\mu_B \sim 24$ MeV, the hot expanding system spends a considerable portion of its dynamics in the region of low $\frac{\eta}{s}$; an optimal situation being, the system stays at low $\frac{\eta}{s}$ and then very quickly freezes out at or close to $T_{\rm cep}$. Such a trajectory would be tantamount to the isobaric trajectory above the critical pressure, shown for H_2O in Fig. 2 (triangles). For other trajectories with μ_B close to, or slightly above $\mu_B^{\rm cep}$, significant collective motion is expected to develop as the system evolves toward freeze-out with higher $\frac{\eta}{s}$ values. Consequently, the dependence of elliptic flow on μ_B is of interest.

Figure 3 shows a differential elliptic flow (v_2) excitation function for charged hadrons, $\langle p_T\rangle = 0.65$ GeV/c, measured in $13 - 26\%$ central Au+Au and Pb+Pb collisions [33]. For the range of collision energies $\sqrt{s_{NN}} \sim 17 - 200$ GeV, Fig. 1 indicates an essentially constant freeze-out temperature $T \sim 165$ MeV for the values $\mu_B \sim 25 - 250$ MeV. Therefore, these v_2 measurements (for $\sqrt{s_{NN}} \sim 17 - 200$ GeV) result from excited systems which all evolve toward our assumed value of $T_{\rm cep}$, albeit with different μ_B values.

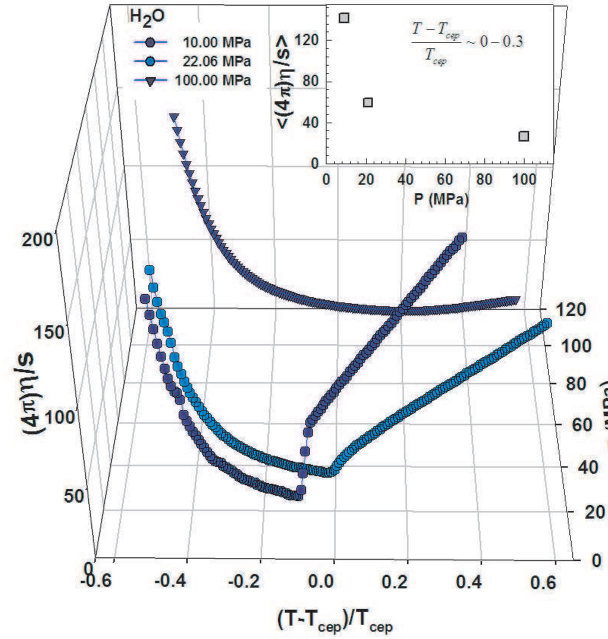

FIG. 2: (Color online) $4\pi(\frac{\eta}{s})$ vs the reduced temperature $\frac{T-T_{\rm cep}}{T_{\rm cep}}$ for H_2O. Results are shown for an isobar at the critical pressure ($P_{\rm cep} = 22.6$ MPa) and one above (below) it, as indicated. The inset shows $\langle 4\pi(\frac{\eta}{s})\rangle$ for the reduced temperature range $0.0-0.3$. The data are taken from Ref. [22].

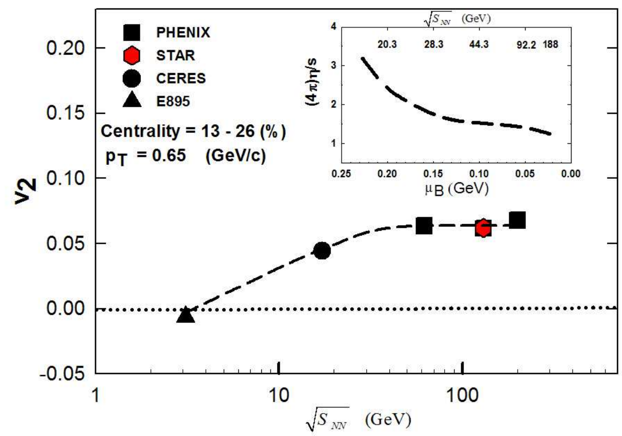

FIG. 3: (Color online) Flow excitation function. The data is obtained from Ref. [33]. The inset shows the schematic variation of $\langle 4\pi(\frac{\eta}{s})\rangle$ vs μ_B.

Figure 3 shows that v_2 is essentially constant for $\sqrt{s_{NN}} \sim 62-200$ GeV. The energy density is estimated to decrease by $\sim$ 30% as the beam collison energy is reduced from $\sqrt{s_{NN}} \sim 200$ GeV to $\sqrt{s_{NN}} \sim 62$ GeV. Therefore we interpret this constancy of v_2 as an indication that (i) the equation of state associated with the crossover transition to the QGP is soft, and (ii) that $\langle\frac{\eta}{s}\rangle$ is relatively small for the μ_B values corresponding to this collision energy range. That is, these μ_B values are significantly smaller than $\mu_B^{\rm cep}$.

For $\sqrt{s_{NN}} \sim 18$ GeV Fig. 3 shows that v_2 decreases by almost 50%, compared to the value for $\sqrt{s_{NN}} \sim 62-200$ GeV. Here, it is important to point out that the mean transverse energy per particle is essentially the same for collision energy range $\sqrt{s_{NN}} \sim 17-200$ GeV and the estimated Bjorken energy density is ~ 5.4 and 3.2 GeV/fm^3 for $\sqrt{s_{NN}} = 200$ GeV [34] and $\sqrt{s_{NN}} = 17$ GeV [35] respectively, i.e the energy density change from $\sqrt{s_{NN}} \sim 62$ GeV to $\sqrt{s_{NN}} \sim 17$ GeV is not very large. Thus, the initial temperature of the high energy density matter created in collisions at $\sqrt{s_{NN}} \sim 62$ GeV and $\sqrt{s_{NN}} \sim 17$ GeV are not drastically different, and the fraction of the elliptic flow generated during the (dissipative) hadronic phase [36] is expected to be qualitatively similar.

A reduction in collision energy from $\sqrt{s_{NN}} \sim 62$ GeV to $\sqrt{s_{NN}} \sim 17$ GeV leads to a significant increase (more than a factor of two) in the value of μ_B. Recent calculations [37] also indicate that, in the hadronic phase, $\frac{\eta}{s}$ decreases with increasing μ_B. Therefore, a significant part of the reduction in v_2 observed as the collision energy is reduced from $\sqrt{s_{NN}} \sim 62$ GeV to $\sqrt{s_{NN}} \sim 18$ GeV, could be a manifestation of the expected increase in $\langle\frac{\eta}{s}\rangle$ for values of $\mu_B \gtrsim \mu_B^{\rm cep}$.

To estimate $\mu_B^{\rm cep}$, we assume a smooth transition in the magnitude of v_2 over the range $62 \lesssim \sqrt{s_{NN}} \gtrsim 18$ GeV, which is not yet measured. The inset in Fig. 3 gives a schematic illustration of the expected change of $\langle 4\pi(\frac{\eta}{s})\rangle$ with μ_B over this collision energy range. We use the "knee" in the extrapolated values for v_2 over the range $62 \lesssim \sqrt{s_{NN}} \gtrsim 18$ GeV to obtain the estimate $\mu_B^{\rm cep} \sim 150-180$ MeV. A similar estimate was obtained by evaluating the μ_B dependence of $\langle\frac{\eta}{s}\rangle$ following the procedures outlined in Refs. [22, 23, 38, 39], followed by interpolation to unmeasured μ_B values.

Given these values of $T_{\rm cep}$ and $\mu_B^{\rm cep}$, we expect the value $\langle 4\pi(\frac{\eta}{s})\rangle$, extracted from flow measurements performed at $\sqrt{s_{NN}} \sim 40$ and 30 GeV, to be significantly larger than those obtained from similar measurements performed over the range $\sqrt{s_{NN}} \sim 62-200$ GeV. This change should also be reflected in the onset of a decrease of v_2, a possible violation of the universal scaling patterns [27, 28, 29] observed for measurements at $\sqrt{s_{NN}} \sim 62-200$ GeV and a measurable increase in v_2 fluctuations.

In summary we have argued that experimental assessment of $\langle\frac{\eta}{s}\rangle$ as a function of μ_B and T provides a good dynamic observable for constraining the critical end point of hot QCD matter. A first estimate for the CEP from flow data indicate the values $T_{\rm cep} \sim 165-170$ and $\mu_B^{\rm cep} \sim 150-180$ MeV. Interestingly, our estimate is in good agreement with the prediction of Gavai et al.

[14], obtained from lattice QCD simulations with realistic (about 1.7 times) pion masses and large volumes. This estimate also places the CEP in the range for direct validation at RHIC via an energy scan. An initial measurement at $\sqrt{s_{NN}} \sim 40$ and 30 GeV would give sufficient information on where to focus more detailed attention.

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